

FSMQ

Additional Mathematics

Unit **6993**: Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation must be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for 6993

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation **isw**. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument. Unless otherwise stated (by for instance, **cao** usually apply **isw**

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation **ft** implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

6993

Mark Scheme

June

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

Question	Answer	Marks	Guidance
1	$-6 < 2x - 1 < 7$ $\Rightarrow -5 < 2x < 8 \Rightarrow -\frac{5}{2} < x < 4$ isw S.C. If solution only given as number line with two clear open circles at ends then B2	B1 B1 B1 [3]	One end point The other Both together - accept 2 separate inequalities linked by "and" cao B3 Final answer seen www Condone incorrect signs (including equals) for first 2 marks. Ignore listings. but not "or" or comma or nothing

Question	Answer	Marks	Guidance
2	$\frac{dy}{dx} = 3x^2 - 4x + 7$ $\Rightarrow y = x^3 - 2x^2 + 7x + c$ oe Substitute (1, 2) $\Rightarrow 2 = 1 - 2 + 7 + c$ $\Rightarrow c = -4$ $\Rightarrow y = x^3 - 2x^2 + 7x - 4$	M1 A1 M1 A1 [4]	Integrate (at least 2 powers increased by 1) Ignore lack of $y =$ and c Dependent on 1st M Final equation must be seen Beware just multiplying by x Must include $y =$ and -4 Condone $f(x) =$

Question		Answer	Marks	Guidance	
3	(i)	$\text{Area} = \int_0^2 8x^3 \, dx = [2x^4]_0^2$ $= 32 - (-0) = 32$	M1 M1 A1 [3]	Integrate correct function (power of 4 soi). Ignore wrong or no limits Substitute $x = 2$. Dependent on first M	Beware just multiplying by x i.e. $8x^4$
	(ii)	Add a rectangle of area 10 \Rightarrow Total area = 42	B1 [1]	ft Addition of 10 must be correct!	Can be by integration

Question		Answer	Marks	Guidance	
4	(i)	$s = 0.09t^2 - 0.0001t^3$ $\Rightarrow v = 0.18t - 0.0003t^2 \text{ isw}$ When $v = 0$, $t = 0$ or $t = \frac{0.18}{0.0003} = 600$ Time = 600 seconds or 10 mins isw	M1 A1 M1 A1 [4]	Diffn (both powers reduced by 1). Set $v = 0$ and any attempt to solve. Dependent on first M. Units required.	Beware division by t . Condone division by t or a constant when expression not set to 0
	(ii)	Substitute <i>their</i> t into s $\Rightarrow s = 10800 \text{ m or } 10.8 \text{ km } \text{ isw}$	M1 A1 [2]	Units required, but only withhold this mark for units wrong or missing if not already withheld in part (i).	

Question		Answer	Marks	Guidance
5	(i)	Angle BAL = $20 + 25 = 45$ Angle ABL = $180 - 65 = 115$ soi OR exterior angle = 65 soi OR angle LBN = 40 soi Angle ALB = 20	B1 B1 B1 [3]	 Or B2 for ALB www
	(ii)	AB = 7 soi $\frac{LB}{\sin 45} = \frac{AB}{\sin 20} \left(= \frac{7}{\sin 20} \right)$ $\Rightarrow LB = \frac{7 \sin 45}{\sin 20} = 14.5$	B1 M1 A1 [3]	For <i>their</i> AB and <i>their</i> angles. Anything that rounds to 14.5

Question		Answer	Marks	Guidance
6	(i)	$f(3) = 0 \Rightarrow 27 - 36 + 3a + b = 0$ or better $f(1) = 4 \Rightarrow 1 - 4 + a + b = 4$ or better Solve <i>their</i> simultaneous eqns from above $\Rightarrow a = 1, b = 6$	B1 B1 M1 A1 [4]	e.g. $3a + b = 9$. e.g. $a + b = 7$ Attempt to find a and b from <i>their</i> eqns
	(ii)	Sight of $(x - 3)(x^2 + px + q)$ for any p, q Or algebraic division $\Rightarrow (x - 3)(x + 1)(x - 2) = 0$ $\Rightarrow (x =) 3, 2, -1$	M1 A1 A1 [3]	Or attempt to find another root of <i>their cubic</i> by remainder theorem soi For correct complete factorisation soi by final answer Correct solution

Question		Answer	Marks	Guidance
7	(i)	$\text{Distance}^2 = 5-3^2 + 11-7^2 \quad (=20)$ $\Rightarrow \text{distance} = \sqrt{20} = 2\sqrt{5}$	M1 A1 [2]	Soi e.g. by 4.47..... Must be exact isw $d = 20$ is M0
	(ii)	Centre = midpoint = (4, 9) Radius = $\sqrt{5}$ or decimal equivalent $\Rightarrow x-4^2 + y-9^2 = 5$ OR $x^2 + y^2 - 8x - 18y + 92 = 0$	B1 B1 B1	Centre Radius soi by for e.g. $r^2 = 5$ ft <i>their</i> identified centre (but don't accept A or B) isw Rhs must be 5, not $\sqrt{5}^2$
		Alternative: $\frac{y-11}{x-5} \cdot \frac{y-7}{x-3} = -1$ $\Rightarrow y-11 \quad y-7 + x-5 \quad x-3 = 0$ $\Rightarrow x^2 + y^2 - 8x - 18y + 92 = 0$	B1 B1 B1 [3]	Use of $m_1 m_2 = -1$

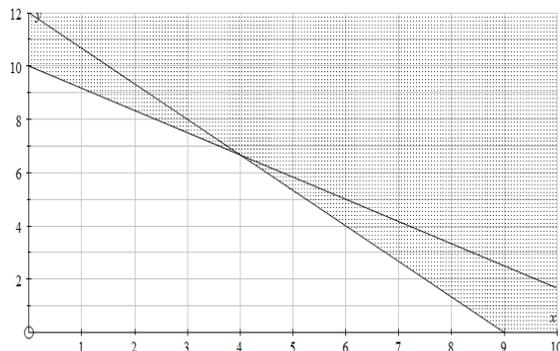
Question		Answer	Marks	Guidance
9	(i)	$\frac{1 - \cos^2 x}{1 - \sin^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$	B1 [1]	Use of $\frac{\sin x}{\cos x} = \tan x$ and use of $\sin^2 x + \cos^2 x = 1$
	(ii)	$\frac{1 - \cos^2 x}{1 - \sin^2 x} = 3 - 2 \tan x$ $\Rightarrow \tan^2 x = 3 - 2 \tan x$ $\Rightarrow \tan^2 x + 2 \tan x - 3 = 0$ $\Rightarrow \tan x + 3 \quad \tan x - 1 = 0$ $\Rightarrow \tan x = -3$ or $\tan x = 1$ $\Rightarrow x = 108(4\dots)$ or 45	M1 M1 A1A1 [4]	Correct use of (i) Factorise their three term quadratic or insertion of <i>their</i> values into correct formula Dep on 1st M -1 for any other values inside range, ignore extra values outside range (Check the two linear factors by whether they multiply out to give the first and last terms of their quadratic.)

Question		Answer	Marks	Guidance
10	(i)	$y = 2x^2 + x - 5 \Rightarrow \left(\frac{dy}{dx} = \right) 4x + 1$ $= 5$ $\Rightarrow x = 1, y = -2$	B1 M1 A1 [3]	Equating to 5 and solving For both
	(ii)	gradient normal = $-\frac{1}{5}$ $\Rightarrow (y + 2) = -\frac{1}{5} x - 1$ $\Rightarrow 5y + x + 9 = 0$	B1 M1 A1 [3]	Gradient of normal soi Equation using <i>their</i> (1, -2) and <i>their</i> normal gradient (which may only be $\pm\frac{1}{5}$ or -5) oe , but only 3 terms isw

Section B

Question	Answer	Marks	Guidance	
11 (i)	Length = $30 - 2x$ Breadth = $14 - 2x$ (Height = x) $\Rightarrow V = (30 - 2x)(14 - 2x)x$ $= 4x^2 - 88x + 420 x$ oe $= 4x^3 - 88x^2 + 420x$	B1 B1 M1 A1 [4]	Soi Soi Product of <i>their</i> length, breadth and x www ; must show at least one product of any two lengths step (N.B. Answer given)	N.B. dimensionally correct e.g. $= (30 - 2x)(14x - 2x^2)$ Length and breadth must be functions of x
(ii)	$V = 4x^3 - 88x^2 + 420x$ $\frac{dV}{dx} \Rightarrow 12x^2 - 176x + 420$ isw $= 0$ when $12x^2 - 176x + 420 = 0$ $\Rightarrow 3x^2 - 44x + 105 = 0$ $\Rightarrow 3x - 35 \quad x - 3 = 0$ $\Rightarrow x = \frac{35}{3}$ or anything that rounds to 11.7, $x = 3$ S.C. Answers only B1, B1	M1 A1 M1 M1 A1 [5]	Diff (at least two powers reduced by 1) Set <i>their function</i> = 0 Dep on 1st M Soi by solution Factorise three term quadratic or insertion of <i>their</i> values into correct formula Both	Beware division by x . Condone prem div by a constant. (Check the two linear factors by whether they multiply out to give the first and last terms of their quadratic.)
(iii)	$x = \frac{35}{3}$ should be rejected as it is over half of 14 Substitute an acceptable x into V ($0 < x < 7$) Volume = 576	B1 M1 A1 [3]	ft from <i>their</i> incorrect x Explanation necessary (e.g. one length is -ve) Alt: use of second derivative acceptable. Ignore units	"V is -ve" as the only explanation not accepted. Nor is "x is too big".

Question		Answer	Marks	Guidance
12	(i)	$\text{Time out} = \frac{15}{x}, \text{Time back} = \frac{15}{x-2}$ $\text{Total time} = \frac{15}{x} + \frac{15}{x-2}$	B1 B1 [2]	For one Addition of two correct terms isw
	(ii)	$\frac{15}{x} + \frac{15}{x-2} = 6$ $\Rightarrow 15x - 2 + 15x = 6x(x-2)$ $15x - 30 + 15x = 6x^2 - 12x \text{ oe}$ $\Rightarrow 6x^2 - 42x + 30 = 0$ $\Rightarrow x^2 - 7x + 5 = 0$	B1 M1 A1 A1 [4]	Equate <i>their</i> time to 6 Multiply throughout by LCM Brackets cleared www At least one interim step must be seen. N.B. Answer given
	(iii)	$x^2 - 7x + 5 = 0$ $\Rightarrow x = \frac{7 \pm \sqrt{49 - 20}}{2} = \frac{7 \pm \sqrt{29}}{2}$ <p>or 6.19 and 0.807 (Paul's speed) = 6.19 km hr⁻¹</p> <p>S.C (Paul's speed) is 6.19 km hr⁻¹ B2 or $x = 6.19$ B1</p>	M1 A1 A1 [3]	Use of correct formula with given equation + units but only if <i>their</i> 0.807 is discarded Condone 0.81 or 0.8 but not 6.2 N.B. If 3 marks not awarded then look for the S.C.
	(iv)	$\frac{15}{x-2} - \frac{15}{x} = \frac{15}{4.19} - \frac{15}{6.19}$ <p>Sight of 3.58 or 2.42 or 1.16 69 or 70 mins or 1hr 9 mins or 1 hr 10 mins</p>	M1 B1 A1 [3]	Sub <i>their</i> x into correct expression Or sub <i>their</i> x into the two separate correct expressions for time and then subtract Answer correct in minutes Or the simplified expression Accept other way round giving a negative value

Question		Answer	Marks	Guidance
13	(i)	$100x + 120y \leq 1200$ oe $2x + 1.5y \leq 18$ oe	M1 A1 A1 [3]	Attempting to use information to create an inequality Ignore extra inequalities Seen by one LH side Condone use of <
	(ii)		B1 B1 B1 B1 [4]	One line (allow intercepts ± 0.1) Other line (allow intercepts ± 0.1) Shading one line – 1 st quad only Shading other line – 1 st quad only N.B. Shading below a line gets B0 N.B. Candidates may get inequalities wrong in (i) but get the shading correct in (ii) - this should be allowed. Allow ft for shading of wrong line but only if the gradient is -ve Allow ft for shading of wrong line but only if gradient is -ve and the two lines intersect in the 1st quadrant (not on axes)
	(iii)	$(P =) 3.5x + 3y$ e.g. (9, 0) gives 31.5, (0, 10) gives 30 (4, 6) gives 32, (5, 5) gives 32.5 (3, 7) gives 31.5 (6, 4) gives 33 (6, 4) gives 33 S.C. for last 4 marks. (6,4) gives 33 B2 Either 33 or (6,4) B1	B1 M1 A1 A1 A1 [5]	Ignore any equating to a number test at least two integer points in correct feasible region in correct OF Both points correct. Ignore any others For (6, 4) chosen For 33 i.e. the point must be identified as the maximum.

Question		Answer	Marks	Guidance	
14	(i)	Probability remains constant Imperfection of mugs independent	B1 B1 [2]	Not "Random"	
	(ii)	$p = \frac{1}{20}, q = \frac{19}{20}$ $P(0 \text{ or } 1) = \left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^9 \left(\frac{1}{20}\right)$ $= 0.5987 + 0.3151$ $= 0.9139$ $P(\geq 2) = 1 - 0.9139$ $= 0.086$	B1 M1 A1 A1 M1 A1 [6]	Soi $q^{10} + kpq^9$ attempted, k an integer > 0 $p + q = 1$ $k = 10$ soi soi accept rounding to 3dp Dependent on previous M (Anything that rounds to 0.086)	For k Not $\binom{10}{1}$ or ${}^{10}C_1$
	(iii)	$P(\text{accepted}) = P(0 \text{ imperfect})$ $+ P(1 \text{ imperfect}) \times P(0 \text{ imperfect})$ $\left(\frac{19}{20}\right)^{10} + 10\left(\frac{19}{20}\right)^9 \left(\frac{1}{20}\right) \left(\frac{19}{20}\right)^{10}$ $= 0.5987 + 0.1887$ $= 0.787(4\dots)$ <p>Alternative: $P(\text{accept})$ $= 1 - (\text{Ans to (ii)} + P(1) \times P(\text{at least } 1))$ $= 1 - (\text{ans to (ii)} + P(1) \times (1 - P(0)))$ $= 1 - (0.0861 + 0.3151 \times 0.4013)$ $= 1 - (0.0861 + 0.1264) = 1 - 0.213$ $= 0.787(4\dots)$</p>	M1 A1 A1 A1 [4] M1 A1 A1 A1	Correct plan soi by both correct terms Correct 1st term (as an expression) soi Correct 2 nd term (as an expression including the 10) soi Accept 0.788 [4] Correct plan soi by both correct terms taken from 1 Ft Using <i>their</i> ans to (ii) 2nd term as an expression soi by final answer	Words only sufficient soi by final answer Anything that rounds to 0.787 or 0.788 Words only sufficient

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